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A Novel Approach for All-to-All Routing in All-optical Hypersquare Torus Network

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ABSTRACT

Wavelength division multiplexing (WDM) optical networks are becoming more attractive due to their unprecedented high bandwidth provisions and reliability over data transmission among nodes. Therefore, it is not uncommon for enterprises to build a datacenter with over thousands of nodes using WDM optical networks. To reach the high speed over optical links, all-optical, i.e., single hop, networks are desirable as there is no overhead on conversions to and from the electronic form compared to multi-hop networks. However, given the number of nodes required, few previous works suggested a topology, e.g., torus, to support all-to-all routing with the minimum number of wavelengths over all-optical networks. In this paper, we address this challenge from a different angle. Specifically, it is possible to build different torus topologies by altering the number of nodes in every dimension, but we first show that the minimum number of wavelengths to satisfy the all-to-all routing over torus is $N/3$, and prove that the necessary and sufficient condition to achieve it is the sides of all dimensions are 3; thus the resultant topology is an n-dimensional hypersquare torus network; then we develop a wavelength assignment to achieve the all-to-all routing over the corresponding n-dimensional hypersquare torus; finally, we consider the fail-over problem in our proposed topology and derive the minimum number of backup wavelengths to mitigate the affected lightpaths thus maintain the gossiping.

Keywords

All-to-All Routing; Hypersquare Torus; Routing and Wavelength Assignment; Optical Network

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1. INTRODUCTION

In wavelength division multiplexing (WDM) networks, distinct laser beams can propagate on a fiber link simultaneously. Therefore, WDM networks can potentially support the gigantic bandwidth for many future throughput-intensive applications. The key technique to transfer bulk data over a wavelength routed all-optical network is to establish a lightpath [1] between a source-destination pair, which might come across multiple consecutive links. In the absence of wavelength converters, whether a lightpath can be established is subject to the wavelength continuity constraint - a lightpath must occupy the same wavelength on all the links it traverses between a source-destination pair.

It is not uncommon for enterprises to build a datacenter with over thousands of nodes using WDM optical networks. To maintain the high speed over bulk data transmission, all-optical networks, aka sing hop networks, is most desirable as there is no overhead on conversions to and from the electronic form compared to multi-hop networks [2], which could deteriorate bandwidth provisions. All-to-all routing, also known as gossiping, is a fundamental problem in such all-optical networks and has been studied by numerous previous works. However, they mainly focus on the problem that finding out the minimum number of wavelengths given a topology, e.g., a fixed side and dimension torus network, thus few ones can suggest a topology, e.g., torus, to support all-to-all routing with the minimum number of wavelengths over all-optical networks given the number of nodes required by enterprises.

In this paper, we address this challenge from a different angle. Specifically, it is possible to build different torus topologies by altering the number of nodes in every dimension, thus we first show that the minimum number of wavelengths to satisfy the all-to-all routing over torus is $N/3$, where N is the number of nodes, and prove that the necessary and sufficient condition to achieve it is that the sides of all dimensions of networks are 3; thus the resultant topology is an n-dimensional hypersquare torus network; then we develop a wavelength assignment to achieve the all-to-all routing over the corresponding n-dimensional hypersquare torus; finally, we address the fail-over problem in our proposed topology and derive the minimum number of backup wavelengths to mitigate the affected lightpaths thus maintain the gossiping.

To the best of our knowledge, no previous work achieves the minimum number of wavelengths over n-dimensional hypersquare torus topologies where dimension is not less than 3 and the side is odd. In summary, our key contributions include:

1. We develop a novel routing and wavelength assignment algorithm using greedy algorithm (GAR) to achieve all-to-all routing in ring networks with odd nodes and even nodes using the minimum number of wavelengths. Although this problem was investigated in [3] and [4], they failed to propose a wavelength assignment algorithm to achieve the optimality.

2. We show that, for any torus topology with N nodes in the network, the minimum number of wavelengths to satisfy all-to-all routing is $N/3$, which can be achieved in the n-dimensional hypersquare torus with side 3.

3. Inspired by (1) we also develop a routing and wavelength assignment algorithm to address the all-to-all routing problem in the n-dimensional hypersquare torus with side 3 with $N/3$ wavelengths, which achieves the theoretical minimum number of wavelengths.

4. We consider the links fail-over problem in the n-dimensional hypersquare torus with side 3 and obtain the lower bound of number of backup wavelengths to maintain the all-to-all routing in the corresponding networks.

The remainder of the paper is organized as follows. Section 2 introduces our GAR algorithm to realize all-to-all routing in a ring network with odd or even nodes. Section 3 shows that the n-dimensional hypersquare torus with side 3 can achieve the minimum number of wavelengths among all torus topologies, and a routing and wavelength assignment algorithm is proposed to achieve that minimum. The fail-over problem in n-dimensional hypersquare tori with side 3 is addressed in Section 4. We review related works in Section 5 and conclude our paper in Section 6.

Table 1: Parameters and notations

Notation	Meaning
n	The dimension of the torus
k	The side of each dimension in hypersquare tori
N	The number of nodes in the topology
m	$\lfloor k/2 \rfloor$
$\oplus$	The addition modulus k plus one
$\ominus$	The subtraction modulus k plus one
G_k^n	The n-dimensional hypersquare torus with side k
l_i	The link between node i and node $i+1$ in ring networks
D_k^n	The average shortest path length in G_k^n
A_k^n	The average number of wavelengths in links in G_k^n
Min_k^n	The lower bound of wavelengths in G_k^n
d_i	The side of the i^{th} dimension of n-dimensional tori
$E[d_i]$	The average shortest path length in d_i
$E[d]$	The average shortest path length in n-dimensional tori
U_N	The minimum number of wavelengths in tori with N nodes
$[a, b]$	$\{a, a+1, \cdots, b\}$
$R(i, j)$	The lightpath from node i to node j
w_{ij}	The wavelength assigned to $R(i, j)$
l_{ij}	The unidirectional link from node i to node j
l_i	The unidirectional link from node i to node $i \oplus 1$ in ring networks

2. ALL-TO-ALL ROUTING OVER ALL-OPTICAL NETWORKS

The all-to-all routing problem over all-optical networks was well studied by numerous previous works but still opens in many ways. Bermond et al. [3] showed that the minimum number of wavelengths needed over a ring network of N nodes, where $N = k$ for ring networks, is $k^2/4 - 1$ if k is odd, and $k^2/4$ if k is even. They also showed that 2^{n-1} wavelengths are required to achieve all-to-all routing over n-dimensional hypercubes. Schroder et al. [17] showed that the minimum number of wavelengths for all-to-all routing over all-optical 2-dimensional torus with side k is $(k^3 - k)/8$. Beauquier [5] derived the minimum number of wavelengths required for the n-dimensional hypersquare with side k is $k^{n+1}/8$ if k is even, but did not solve the case for k is odd. To the best of our knowledge, no achievable lower-bound on the number of wavelengths is given when the dimension of torus is no less 3, i.e., $n \geq 3$, and the side k is odd. Additionally, previous works failed to propose a routing and wavelength assignment (RWA) algorithm to achieve those lower-bounds, hence we develop a novel RWA algorithm using greedy algorithm (GAR) to achieve the all-to-all routing over ring networks with both odd and even nodes, which can be applied readily in practical.

2.1 Preliminaries

Fig. 1 depicts a undirected ring with N nodes and N links, where $N = k$. Let l_i denote the link connecting node i and node $i \oplus 1$, where $\oplus$ denotes the addition modulus k then plus one [3]. Every link is bidirectional and every node is required to establish a lightpath with any other node of the ring. As ring is symmetric, node i, where $i \leq N$, is the center node, i.e., locates at the center of the ring, when node i is connecting to all other nodes through $k-1$ lightpaths. Without loss of generality, we assign wavelengths for the (k-1) lightpaths of node i sequentially from $i = 1$. Let $R(i, j)$ denote the lightpath from node i to node j. $R(i, j)$ is in the progressive direction if $(j \ominus i) < m$, where $m = \lfloor k/2 \rfloor$, and regressive direction if $(j \ominus i) > m$ as shown in Fig. 2, where $\ominus$ denotes the operation of subtraction modulus k plus one. The shortest path routing algorithm is adopted in this model [3, 5, 6], thus the set of paths is unique if k is odd, and there are two shortest paths from node i to node $i \oplus k/2$ if k is even. Let $S_{i,j}$ denote the set of wavelengths assigned to l_j when node i is the center node. Note that all the parameters and notations used are shown in Table 1.

To introduce GAR we first define some notations.

DEFINITION 1. $[i, i \oplus m] = \{i, i \oplus 1, \cdots, i \oplus m\}$

DEFINITION 2. *We say $i \leq a \leq i \oplus m$ if the range of a is*

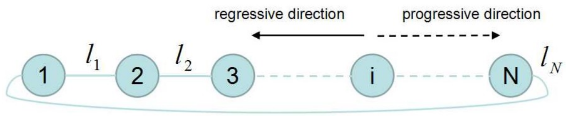

Figure 1: Ring network with N nodes

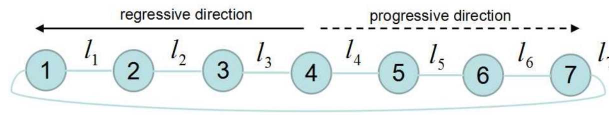

Figure 2: Ring network with 7 nodes

$[i, i \oplus m]$

When k is odd, we have the following lemmas.

LEMMA 1. *$l_{i\oplus m}$ won't be used if node i is the center node, i.e.,*

$$|S_{i,i\oplus m}| = 0 \tag{1}$$

PROOF. When node i is the center node, the m lightpaths in the progressive direction will traverse the m links $l_i, l_{i\oplus 1}, \cdots, l_{i\oplus(m-1)}$ and the m regressive lightpaths will travel the m links $l_{i\ominus 1}, l_{i\ominus 2}, \cdots, l_{i\ominus m}$ with the shortest path routing algorithm. Hence, $l_{i\oplus m}$ is not used. $\square$

LEMMA 2. *$\forall$ a, b, satisfy $i \leq a < b \leq i \oplus m$, we have:*

$$S_{i,b} \subset S_{i,a}, |S_{i,a}| = m + i - a \tag{2}$$

$\forall$ a, b, satisfy $i \oplus (m+1) \leq a < b < i \oplus m$, we have:

$$S_{i,a} \subset S_{i,b}, |S_{i,a}| = a - (i + m) \tag{3}$$

PROOF. Assume that $i \leq a < b \leq i \oplus m$. $\forall w \in S_{i,b}$, suppose that w is the wavelength assigned to $R(i,c)$, where $b \leq c \leq i \oplus m$. $R(i,c)$ will also traverse l_a because of the shortest path routing algorithm, i.e., $w \in S_{i,a}$. Hence, $S_{i,b} \subset S_{i,a}$. Furthermore, $R(i,e)$, where $a+1 \leq e \leq i \oplus m$, will travel on l_a. Therefore, $|S_{i,a}| = m+i-a$. In the same token, we obtain the results for $i \oplus (m+1) \leq a < b < i \oplus k$. $\square$

Let G_k^n denote the n-dimensional hypersquare torus with side k thus G_k^1 is a ring network with k nodes. We first introduce the theoretical lower-bound of the wavelengths needed for all-to-all routing in all-optical G_n^k.

2.2 Lower-bound Wavelengths

M. A. Marsan et al. [7] derived the average shortest path length in G_k^n, denoted by D_k^n, as follows:

$$D_k^n = \begin{cases} \frac{n(k^{n+1}-k^{n-1})}{4(k^n-1)} & k \text{ is odd} \\ \frac{nk^{n+1}}{4(k^n-1)} & k \text{ is even} \end{cases} \tag{4}$$

Since all links along a lightpath share a same wavelength, the average number of a wavelength assigned to a lightpath is also D_k^n.

As the number of nodes in G_k^n is k^n, thus $N = k^n$, there are $N(N-1)$ ordered node pairs, i.e., $N(N-1)$ lightpaths, and nN links in G_k^n. Let A_k^n denote the average number of wavelengths over one link in G_n^k, we have:

$$A_k^n = \frac{N(N-1)D_k^n}{nN} \tag{5}$$

Substitute (4) into (5), we have:

$$A_k^n = \begin{cases} \frac{N}{4}(k - \frac{1}{k}) & k \text{ is odd} \\ \frac{Nk}{4} & k \text{ is even} \end{cases} \tag{6}$$

Let Min_k^n denote the lower-bound of wavelengths needed for all-to-all routing in G_n^k, thus we have

$$Min_k^n = \lceil \frac{1}{2} A_k^n \rceil \tag{7}$$

which is proved in [5]. Substitute (6) into (7) we have:

$$Min_k^n = \begin{cases} \frac{N}{8}(k - \frac{1}{k}) & k \text{ is odd} \\ \lceil \frac{Nk}{8} \rceil & k \text{ is even} \end{cases} \tag{8}$$

Next we propose a RWA algorithm that can achieve (8) in ring networks by considering two cases: (1) k is odd and (2) k is even, in the following section.

2.3 RWA Algorithm in Ring Networks

Case 1: k is odd:

We propose a greedy algorithm to achieve (8). $N = k = 2m+1$ if k is odd in a ring network. As shown in Fig. 2, node i is randomly chosen to be the center node, and we assign wavelengths to $R(i,j)$ and $R(i, 2i \ominus j)$. As ring networks are symmetric at every node, the wavelength assigned to $R(i,j)$ can be the same as $R(i, 2i \ominus j)$. Thus GAR only needs to achieve (8) in ether $R(i,j)$ or $R(i, 2i \ominus j)$. We first introduce some notations:

DEFINITION 3. *$[a,b] = \{a, a+1, \cdots, b\}$, where a and b are positive integers and $1 \leq a < b$.*

The crux of GAR is as follows: given the set of available wavelengths for a link, we assign a wavelength with the smallest index to a lightpath without contending with the wavelengths of the links along that lightpath. Let N denote the number of nodes in a ring network, $[1, Min_N^1]$ denote the set of available wavelengths, which has the minimum size as shown in (8). Let w_{ij} denote the wavelength assigned to $R(i,j)$. GAR is shown in the pseudo-code in Algorithm 1, which can be summarized into the following four ways:

(i) the available wavelengths in the N links are initialized with $[1, Min_N^1]$. $availWaves[\mu]$, where $1 \leq \mu \leq N$, denotes the available wavelengths in l_μ. (Line 2)

(ii) node 1 is first centered and it needs to build $R(1,j)$, where $1 < j \leq m+1$, thus links $l_1, l_2 ..., l_{m-1}$ will be used. As shown in Lemma 2, l_1 needs to carry m wavelengths, and l_γ, where $1 < \gamma \leq m$, carries the subset of wavelengths of the l_1. With greedy algorithm, the first $m - (\gamma \ominus 1)$ wavelengths in $S_{1,1}$ are assigned to l_γ to build $R(1,j)$, where $2 < j \leq m+1$.

(iii) the N nodes are in turn to be the center nodes starting from 1. Let i denote the index of the node that becomes the center node currently. Similar to (i), l_i needs to carry m wavelengths as well and l_μ, where $i \leq \mu < m \oplus i$, carries $m - (\mu \ominus i)$ wavelengths, which is the subset of l_i. $S_{i,i}$ is assigned the first m available wavelengths in l_i. If $i < \mu \leq N$, the first $m - (\mu \ominus i)$ wavelengths in $S_{i,\mu\ominus 1}$ are assigned to $S_{i,\mu}$. If $1 \leq \mu < i \oplus m$, the first wavelength in $S_{i,\mu\ominus 1}$ was assigned to l_μ before, thus the first $m - (\mu \ominus i) + 1$ available wavelengths except for the first one are assigned to l_μ. (Lines 3-17)

Algorithm 1 GAR in ring with odd nodes

```
1: m = ⌊N/2⌋
2: availWaves = InitWavelengths(N)
3: for each i in [1, N] do
4:     S_{i,i} = availWaves[i][1 : m]
5:     availWaves[i] = availWaves[i] − S_{i,i}
6:     for each offset in [1, m − 1] do
7:         j = (i + offset) mod N + 1
8:         num = m − offset
9:         if i + offset ≤ N then
10:            S_{i,j} = S_{i,j⊖1}[1 : num]
11:        else
12:            S_{i,j} = S_{i,j⊖1}[2 : num + 1]
13:        end if
14:        availWaves[j] = availWaves[j] − S_{i,j}
15:        Sort availWaves[j] ascendingly
16:    end for
17: end for
18: for each i in [1, N] do
19:    for each d in [i + 1, i ⊕ m] do
20:        w_{i,d} = S_{i,d⊖1} − S_{i,d}
21:    end for
22: end for
```

(iv) the set of wavelengths in $S_{i,\mu}$, where $1 \le i, \mu \le N$, is converted to $w_{i,j}$, where $i < j \le i \oplus m$. (Lines 18-22)

For example, GAR for G_7^1 is shown in Table 2. The set of available wavelengths to support all-to-all routing in G_7^1 is $[1, 6]$ as $Min_7^1 = 6$. When node 1 is centered, by (ii), GAR assigns $\{1, 2, 3\}$ to $S_{1,1}$, $\{1, 2\}$ to $S_{1,2}$ and $\{1\}$ to $S_{1,3}$. When node 2 is centered, by (iii), $S_{2,2} = \{3, 4, 5\}$, $S_{2,3} = \{3, 4\}$ and $S_{2,4} = \{3\}$. When node 6 is centered, $S_{6,7} = \{2, 4\}$. $S_{6,1} = \{4\}$ as wavelength 2 was assigned to $S_{1,1}$.

We will prove the correctness of GAR. Let $W_{i,\mu}$ denote the set of wavelengths assigned to l_μ when the first i nodes are the centers and $S_{i,\mu}[e]$ the e^{th} element in $S_{i,\mu}$.

Theorem 1. *$\forall 1 \le \mu \le N$, GAR guarantees that*
(i) $\forall 1 \le a < b \le N$, we have

$$S_{a,\mu} \cap S_{b,\mu} = \emptyset \tag{9}$$

(ii) $\bigcup_{i=1}^{N} S_{i,\mu} = [1, Min_N^1]$

Before proving *Theorem* 1, we first introduce some lemmas.

Lemma 3. *$\forall 1 \le \mu \le N$, GAR ensures that*

$$\sum_{i=1}^{N} |S_{i,\mu}| = Min_N^1 \tag{10}$$

Proof. When node μ is the center, we have $|S_{\mu,\mu}| = m$. $\forall \mu \le i \le \mu \oplus m$, $|S_{i,\mu}| = m - (i \ominus \mu)$. If $\mu \oplus m < i < \mu$, $|S_{i,\mu}| = 0$. Hence, we have

$$\sum_{i=1}^{N} |S_{i,\mu}| = N^2/8 - 1 = Min_N^1 \tag{11}$$

Furthermore, we have

$$|W_{i,i}| = Min_N^1 \tag{12}$$

where $m \le i \le N$. □

Table 2: An example of GAR for G_7^1

μ \ i	1	2	3	4	5	6	7
1	1,2,3					4	5,6
2	1,2	3,4,5					6
3	1	3,4	2,5,6				
4		3	2,5	1,4,6			
5			2	1,4	3,5,6		
6				1	3,5	2,4,6	
7					3	2,4	1,5,6

Lemma 4. *When the first m nodes are the centers, if $\forall 1 \le \mu \le N$ and $\forall 1 \le a < b \le m$, we have*

$$S_{a,\mu} \cap S_{b,\mu} = \emptyset \tag{13}$$

Proof. If $1 \le j \le m$, with the greedy algorithm we have

$$S_{i,i}[j] = \begin{cases} S_{j,j}[j + m - i] & 1 \le j < i \le m \\ (m - \frac{i-2}{2})(i-1) + j - i + 1 & i \le j \le m \end{cases} \tag{14}$$

and

$$W_{j,j} = [1, (m - (i-1)/2)i] \tag{15}$$

If $j = m$, $W_{m,m} = [1, Min_N^1]$, thus there is no wavelength contention in the first m links. According to Lemma 2, $\forall m \le a < b \le 2m - 1$, we have

$$W_{m,b} \subset W_{m,a} \subseteq W_{m,m} \tag{16}$$

which indicates that no wavelength conflicts in l_μ, where $m + 1 \le \mu \le 2m - 1$. Therefore, no wavelength contention occurs in the N links, i.e.,

$$S_{a,\mu} \cap S_{b,\mu} = \emptyset, \tag{17}$$

when the first m nodes are the center nodes, where $1 \le \mu \le N$ and $1 \le a < b \le m$. □

Lemma 5. *If $m < i \le N$, we have*

$$W_{i,i} = [1, Min_N^1] \tag{18}$$

Proof. With the greedy algorithm, when node i is the center, $S_{i,i}[e]$ cannot appear in $S_{i,i+m-e+1}$ where $1 \le i, e \le m$, thus $S_{i,i}[e]$ can be an element of $S_{t,t}$, where $t = i - m + e - 1$. If node j, where $j < t$, is centered, GAR guarantees that no wavelength contention occurring in l_μ, where $j \le \mu \le N$, we have

$$S_{t,t} = \{w | w = S_{t-m+x,t-m+x}[x+1], 0 \le x \le m-1\} \tag{19}$$

Since GAR ensures no wavelength contention occurs when the first m nodes are the centers as shown in *Lemma* 4, we can assign m elements from the corresponding $S_{t-m+x,t-m+x}$ respectively to $S_{t,t}$ by iteration. According to *Lemma* 3, if $m \le i \le N$, we have $|W_{i,i}| = Min_N^1$, hence

$$W_{i,i} = [1, Min_N^1] \tag{20}$$

where $m < i \le N$. □

With *Lemmas* 4, 5, we prove *Theorem* 1 as follows:

Proof. According to *Lemma* 4 and 5, if node i is centered, GAR ensures that no wavelength contention occurs in l_μ, where $i \le \mu \le N$. As shown in *Lemma* 4 we have

$$W_{i,a} \subset W_{i,b} \tag{21}$$

where $i \le b < a \le N$. As the m wavelengths in $S_{i,i}$ are not included in $W_{i-1,i}$, the first $m-(a-i)+1$ wavelengths in $S_{i,a}$ do not conflict with the wavelengths in $W_{i-1,a+1}$. Thus we have

$$S_{i,a}[e] = S_{i,i}[e] \tag{22}$$

where $1 \le e \le m-(a-i)+1$.

If $1 \le \mu < i$, the first element in $S_{i,\mu}$ is in $W_{i-1,\mu\oplus 1}$. If $m+3 \le i \le N$ and $1 \le \mu \le i \ominus (m-1)$, we have

$$S_{i,N}[\mu] = S_{i,i}[\mu] = S_{i-m+\mu-1,i-m+\mu-1}[\mu] \tag{23}$$

By iteration we obtain that

$$S_{i,N}[\mu] \in W_{i-1,\mu} \tag{24}$$

and when $1 \le e < \mu$, there is

$$S_{i,N}[e] \notin W_{i-1,\mu} \tag{25}$$

If node i, where $m+1 \le i \le N$, is centered and $0 \le \mu \le i-m-2$ (l_0 is l_N), we have

$$S_{i,N}[e] \notin W_{i-1,\mu} \tag{26}$$

where $\mu+1 \le e \le i-m-\mu-1$. Except for the first element, all other elements in $S_{i,\mu}$ are not in $W_{i-1,\mu+1}$, thus we have

$$S_{i,\mu+1} = \{w | w = S_{i,\mu}[x], 2 \le x \le i-m-\mu-1\} \tag{27}$$

Therefore, we have

$$W_{N,\mu} = [1, Min_N^1] \tag{28}$$

where $1 \le \mu < m$, and

$$S_{a,\mu} \cap S_{b,\mu} = \emptyset \tag{29}$$

where $1 \le \mu \le N$ and $1 \le a < b \le N$. We combine (18),(28) and obtain

$$W_{N,\mu} = [1, Min_N^1] \tag{30}$$

where $1 \le \mu \le N$. $\square$

Therefore, GAR can optimally realize all-to-all routing in an all-optical ring networks with N nodes.

Case 2: *k* is even

In a ring network with k nodes, where k is odd, each progressive lightpath has its counterpart in the regressive direction, but when k is even, i.e., $k = 2m$, $R(i, i \oplus m)$ has no counterpart in another direction. We summarize our GAR algorithm in this case into 2 ways: first, we satisfy all the lightpaths with the minimal number of wavelengths except those special ones that have no counterparts, and second assign wavelengths for them.

(i) the algorithm is the same as that in Case 1. Similarly, when node i is the center node, we assign the same wavelength for $R(i,j)$ and $R(i, 2i \ominus j)$, where $i < j < i \oplus m$, and only consider the $k(k-2)/2$ lightpaths in the progressive direction. The available set of wavelengths in either direction is $\{1, 2, \cdots, Min_k^1\}$, where $Min_k^1 = \lceil k^2/8 \rceil$. With greedy algorithm, $k^2/8 - k/4$ wavelengths are enough to satisfy the lightpaths in the progressive direction. The proof is the same as that in Case 1.

(ii) We only need to assign wavelengths for the k lightpaths of which path lengths are m. There are $\lceil k/4 \rceil$ lightpaths left after (i). If the directions of $R(i, i+m)$ and $R(i+m, i)$, where $1 \le i \le m$, are the same, e.g., progressive or regressive, they form a closed circle. Specifically, the direction is progressive if i is odd, and regressive if i is even,

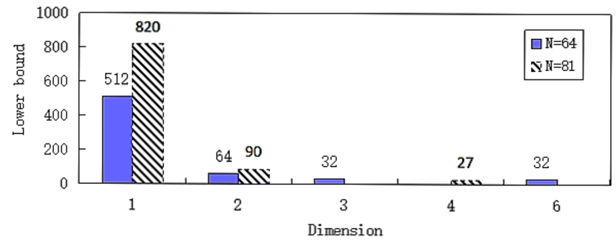

Figure 3: Lower bound of wavelengths when N is fixed

thus the number of wavelengths needed in the progressive direction is $\lceil k/4 \rceil$ and in the regressive direction is $\lfloor k/4 \rfloor$.

In summary, the number of wavelengths needed to achieve (8) in Case 2 is $Min_k^1 = \lceil k^2/8 \rceil$.

3. THE N-DIMENSIONAL HYPERSQUARE TORUS WITH SIDE 3

It is not uncommon fo enterprises to build a cloud/datacenter with fixed number of nodes. A ring network with our GAR algorithm can be used to satisfy the enterprises' requirements. However, when the number of nodes in the system is fixed, the minimum number of wavelengths needed to support all-to-all routing decreases with the increase in the number of the dimensions of torus topologies. Fig. 3 shows the minimum number of the wavelengths needed to support all-to-all routing over topologies with different dimensions when $N = 64$ and $N = 81$ respectively. The number of wavelengths significantly decreases from ring networks to n-dimensional hypersquare torus networks. Therefore, in this section we will find a torus topology with fixed number of nodes that requires the minimum number of wavelengths to support all-to-all routing.

3.1 Lower-bound Wavelengths in Torus Networks with Fixed Nodes

According to (8), $Min_2^n = N/4$ if the side of a hypersquare torus is 2. However, this n-dimensional hypersquare torus with side 2 is an n-dimensional hypercube, thus the number of links in the hypercube is $nN/2$ hence Min_2^n increases from $N/4$ to $N/2$, which is consistent with the results in [3].

Let d_i denote the side of the i^{th} dimension of n-dimension torus networks, where $d_i \ge 2$. If $d_i = 2$, one bidirectional link is sufficient to connect any two nodes in rings along the i^{th} dimension instead of two bidirectional links [8], thus the number of links in rings will be halved, resulting in the double of the number of wavelengths needed over these links. Therefore, $d_i = 2$ is excluded thus we assume $d_i \ge 3$.

THEOREM 2. *The lower-bound wavelengths to realize all-to-all routing in tori with N nodes is $N/3$.*

PROOF. The number of nodes in n-dimensional tori is

$$N = \prod_{i=1}^{n} d_i \tag{31}$$

where $d_i \ge 3$. Without loss of generality, we suppose that d_i is odd when $1 \le i \le s$, and even for $s < i \le n$, where $1 \le s \le n$. Let $E[d_i]$ denote the mean distance along the i^{th} dimension and $E[d]$ the average shortest path length in

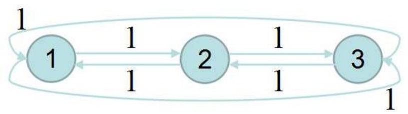

Figure 4: The ring network with 3 nodes

n-dimensional tori. Marsan et al. [7] presented that

$$E[d] = \frac{N}{N-1}\sum_{i=1}^{n} E[d_i] \tag{32}$$

and in the i^{th} dimension, there is

$$E[d_i] = \begin{cases} d_i/4 & d_i \text{ is even} \\ d_i/4 - 1/(4d_i) & d_i \text{ is odd} \end{cases} \tag{33}$$

The number of unidirectional links in the whole network is $2nN$. Let U_N denote the lower-bound wavelengths for all-to-all communication in all-optical torus networks with N nodes, thus we have

$$\begin{aligned} U_N &= \frac{N(N-1)E[d]}{2nN} \\ &= \frac{N}{8n}(\sum_{i=1}^{n} d_i - \sum_{i=1}^{s}\frac{1}{d_i}) \\ &\geq \frac{N}{8n}\sum_{i=1}^{n}(d_i - \frac{1}{d_i}) \\ &\geq \frac{N}{3} \end{aligned} \tag{34}$$

U_N achieves its minimum $N/3$ when all $d_i = 3$. □

Next we develop a RWA algorithm to satisfy the all-to-all routing in G_3^n using the minimum number of wavelengths, i.e., $N/3$.

3.2 Optimal RWA Algorithm for G_3^n

Let $(a_1, a_2, \cdots, a_n)$ denote the location of node p in G_3^n, where $1 \leq a_i \leq 3$, and $p[e]$, where $1 \leq e \leq n$, denote the coordinate of node p in the e^{th} dimension; $l_{i,j}$ denote the unidirectional link from node i to node j. We can regard G_3^n as 3 G_3^{n-1}: $g_1(n-1)$, $g_2(n-1)$ and $g_3(n-1)$. For any node p in $g_h(n-1)$, where $1 \leq h \leq 3$, $p[n] = h$.

Definition 4. *p_h is the counterpart of node p in $g_h(n-1)$ if $p_h[e] = p[e]$, where $1 \leq e \leq n-1$, and $p_h[n] = h \neq p[n]$.*

Let $P(n, h, p)$ denote the set of lightpaths in G_3^n of which sources are the 3^{n-1} nodes in $g_h(n-1)$ and destinations are node p that is not in $g_h(n-1)$; $D(n, p)$ denote the set of $3^{n-1} - 1$ lightpaths in G_3^n whose destinations are node p.

The routing algorithm adopted in G_3^n is to route in the order of dimensions. Before introducing the algorithm to realizing all-to-all routing in G_3^n with 3^{n-1} wavelengths, we first introduce some lemmas.

Lemma 6. *Given a node p in G_3^n, $D(n,p)$ uses one of the $N/3$ wavelengths twice and the left $N/3-1$ wavelengths three times respectively.*

Proof. We proved *Lemma* 6 by induction. □

Lemma 7. *Assume node q is in $g_h(n-1)$, where $1 \leq h \leq 3$. If and only if $R(q,p) \in P(n,h,p)$, $R(q,p)$ will traverse $l_{p_h,p}$.*

Proof. If $R(q,p) \in P(n,h,p)$, with the routing algorithm, $R(q,p)$ will traverse p_h that is the counterpart of node p in $g_h(n-1)$. Then it will traverse $l_{p_h,p}$. If $R(q,p)$ traverses $l_{p_h,p}$, with the routing algorithm, node p is the destination thus node p is not in $g_h(n-1)$ and $R(q,p) \in P(2,h,p)$. □

Lemma 8. *Given two distinct nodes p and q in $g_h(n-1)$, where $1 \leq h \leq 3$. $R(q,p)$, $R(q,p_{h\oplus 1})$ and $R(q,p_{h\ominus 1})$ will share all the links traversed by $R(q,p)$, which are in $g_h(n-1)$.*

Proof. According to the routing algorithm, the first $(n-1)$ steps for $R(q,p_{h\oplus 1})$ and $R(q,p_{h\ominus 1})$ is to route to p thus they will traverse the links travelled by $R(q,p)$. The last step for the two lightpaths is the same as that of $R(p,p_{h\oplus 1})$ and $R(p,p_{h\ominus 1})$, respectively, which are disjoint [9]. □

Lemma 9. *Assume 3^{n-2} wavelengths are sufficient to realize all-to-all routing in G_3^{n-1}. The number of wavelengths in any unidirectional link in the first $n-1$ dimensions to support all-to-all routing in G_3^n is 3^{n-1}.*

Proof. We randomly pick two distinct nodes q and p in G_3^{n-1} and $w_{q,p} = \lambda \in [1, 3^{n-2}]$. G_3^{n-1} is equivalent to $g_h(n-1)$ in G_3^n thus $R(q,p)$ has two counterparts in G_3^n, i.e., $R(q,p_{h\oplus 1})$ and $R(q,p_{h\ominus 1})$. Wavelength λ is mapped to $[3\lambda-2, 3\lambda]$ and we assign the three wavelengths to $R(q,p)$, $R(q,p_{h\oplus 1})$ and $R(q,p_{h\ominus 1})$. According to *Lemma* 8, wavelength λ will be mapped to $[3\lambda-2, 3\lambda]$ in all the links traversed by $R(q,p)$. If we consider all the $R(q,p)$ in $g_1(n-1)$, $g_2(n-1)$ and $g_3(n-1)$ as well as their counterparts, all the lightpaths in G_3^n are considered. Therefore, the set of wavelengths in all the unidirectional links in $g_1(n-1)$, $g_2(n-1)$ and $g_3(n-1)$ are mapped from $[1, 3^{n-2}]$ to $[1, 3^{n-1}]$. □

The algorithm to realize all-to-all routing in G_3^n can be summarized into the following three ways:

(i) all-to-all routing is realized in G_3^1 with one wavelength, as shown in Fig. 4.

(ii) all-to-all routing is realized in G_3^2 with 3 wavelengths. G_3^2 is shown in Fig. 5 and the links connecting the top nodes and the bottom nodes in vertical rings are not depicted. The routing algorithm in G_3^2 is X-Y routing. The wavelength assigned to $R(q,p)$ is

$$w_{q,p} = (q[1] + q[2] + [p[1] + p[2]) \mod 3 + 1 \tag{35}$$

In addition, *Lemma* 6 is established in G_3^2.

(iii) we assume that all-to-all routing is realized in G_3^{n-1} with 3^{n-2} wavelengths and *Lemma* 6 is established. We randomly pick two distinct nodes q and p in G_3^{n-1} and suppose $w_{q,p} = \lambda \in [1, 3^{n-2}]$. We regard G_3^{n-1} as $g_h(n-1)$ in G_3^n, then node p will have two counterparts $p_{h\oplus 1}$ and $p_{h\ominus 1}$. Wavelength λ is mapped to $[3\lambda-2, 3\lambda]$. If wavelength λ is assigned to three lightpaths in $D(n-1,p)$ (*Lemma* 6) and the other two lightpaths are $R(q',p)$, $R(q'',p)$, we assign the three wavelengths to $R_{q,p_{h\oplus 1}}$, $R_{q',p_{h\oplus 1}}$ and $R_{q'',p_{h\oplus 1}}$ and do not violate *Lemma* 9. Similarly, if wavelength λ is assigned to three lightpaths in $D(n-1,p)$ and the other lightpath is $R(q',p)$, we assign wavelengths $3\lambda-2$ and $3\lambda-1$ to $R_{q,p_{h\oplus 1}}$, $R_{q',p_{h\oplus 1}}$, respectively.

We will prove the correctness of this algorithm.

Proof. According to *Lemma* 8, $l_{p,p_{h\oplus 1}}$ carries 3^{n-1} wavelengths. By (iii), $w_{q,p_{h\oplus 1}}$, $w_{q',p_{h\oplus 1}}$ and $w_{q'',p_{h\oplus 1}}$ are in $[3\lambda-$

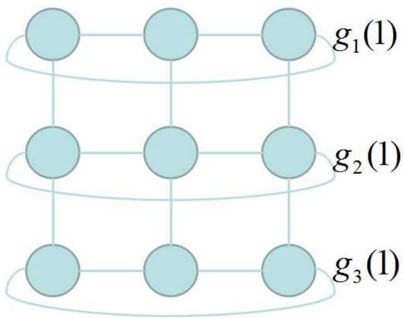

Figure 5: The ring network with 3 nodes

$2, 3\lambda]$. We assign the three wavelengths to the three lightpaths thus the set of wavelengths in $l_{p,p_{h\oplus 1}}$ is $[3\lambda-2, 3\lambda]$. If we consider all the wavelength λ in $[1, 3^{n-2}]$, the set of wavelengths in $l_{p,p_{h\oplus 1}}$ will be $[1, 3^{n-1}]$. With *Lemma* 9, the set of wavelengths in any link in G_3^n is $[1, 3^{n-1}]$. Furthermore, if we fix the destination node $p_{h\oplus 1}$, the 3^n-1 sources are in $g_1(n-1)$, $g_2(n-2)$ and $g_3(n-1)$ thus each wavelength in $[1, N/3]$ will appear 3 times except that one wavelength will appear twice because it is unnecessary to assign wavelength to $R(p_{h\oplus 1}, p_{h\oplus 1})$. Therefore, *Lemma* 6 is still established in G_3^n. $\square$

With the similar approach, we can obtain the RWA algorithm for hypercube with the theoretical minimal wavelengths.

4. LINK FAILURES

All theoretical bounds on the number of wavelengths for all-to-all routing problems are based on the assumption that no link/node failure occurs. In the above discussion, the routing and wavelength assigned to any lightpath is preset, thus all the wavelength assignments can be applied before building the all-optical datacenter/cloud network. However, it is inevitable for a datacenter/cloud to address the link/node failures as they occur frequently after the network is built [10, 11, 12]. The link/node failures indicate that some lightpaths cannot be used, thus an intuitive way to address this problem is to increase the number of wavelengths in the network, thus failed lightpaths can be routed through them immediately. We also assume the number of nodes in networks, i.e., N, is fixed. As shown in Section 3, an n-dimensional hypersquare torus with side 3 is the optimal topology in terms of the number of wavelengths needed to support all-to-all routing. Therefore, in this section we will investigate the problem that how many additional wavelengths are needed to support all-to-all routing if one or multiple links failures occur over an n-dimensional hypersquare torus with side 3.

4.1 One Link Failure

We first address a special case that how many additional wavelengths are required to maintain the all-to-all routing if only one link fails. Let l_{ij} denote the failed unidirectional link in the t^{th} dimension in G_3^n, $(a_1, \cdots, a_{t-1}, s_1, a_{t+1}, \cdots, a_n)$ and $(a_1, \cdots, a_{t-1}, s_2, a_{t+1}, \cdots, a_n)$ denote the coordinates of node i and node j, where $s_1 \neq s_2$.

The number of wavelengths in l_{ij} is $N/3$, thus there are $N/3$ lightpaths passing through this link and we have the following lemma:

LEMMA 10. *Any lightpath originated from node p $(x_1, \cdots, x_t, s_1, a_{t+2}, \cdots, a_n)$ to node q $(a_1, \cdots, a_t, s_2, y_{t+2}, \cdots, y_n)$ will traverse l_{ij}.*

PROOF. The routing algorithm shown in Section 3 first routes in the 1^{st} dimension, then 2^{nd} dimension and so on. From node p to node q, the lightpath first arrives at node i, then passes through l_{ij}. $\square$

There are 3^t choices for node p and 3^{n-t-1} choices for node q, thus $3^t \times 3^{n-t-1} = 3^{n-1}$ lightpaths will traverse l_{ij}. As l_{ij} fails, 3^{n-1} connecting lightpath should be reassigned by additional wavelengths. There are two methods to solve this problem: (1) to reclaim all the wavelengths assigned to the all-to-all routing and reassign the path and wavelength for them. However, as one link fails, the symmetry property of a torus is not valid, thus the wavelength assignment algorithm shown in Section 3 cannot be used. We can apply integer linear programming (ILP) or linear programming (LP) [13, 14, 15, 16] to tackle this RWA problem; and (2) to keep the unaffected lightpaths and only reassign wavelengths to the $N/3$ lightpaths passing through the failure link.

In general, the number of wavelengths solved by ILP and LP is closed to the optimality. However, there are two limitations related to this approach: (1) When a link fails, it is desirable to recover the all-to-all routing as soon as possible, but the convergence period of ILP and LP can be significantly large, especially when N is large; and (2) reconfiguring the optical network is arduous as it requires the pause of network operations. Therefore, in order to reduce the recovery time and reconfiguration cost, we choose the second method, which causes some additional wavelengths.

The fail-over problem is simplified in the second approach compared to ILP and LP. We define a link is the starting (ending) link of node v if this link starts from (ends at) node v. Any node in n-dimensional tori has $2n$ unidirectional starting links and $2n$ unidirectional ending links.

If the failed link is from the ending links of the destination node q, there are 3^t lightpaths destined to this node among the affected 3^{n-1} lightpaths because of the link failure. Let $\alpha(q)$ and $\beta(q)$ denote the set of the $2t$ (or $2t-1$) links in the first t dimensions and the set of the rest of $2(n-t)$ links in the rest of $(n-t)$ dimensions respectively, when node q is the destination. For any two distinct nodes q and p, $\alpha(q)$, $\beta(q)$, $\alpha(p)$ and $\beta(p)$ are disjoint with each other.

Let A and B denote the union of α and β of all 3^{n-t} destination nodes respectively. Each lightpath of the 3^{n-1} affected lightpaths has to traverse one of the links in A to reach its destination. Let C_A and C_B denote the number of lightpaths passing through A and B respectively, thus we have

$$\begin{aligned} C_A &= 3^{n-1} \\ C_B &\geq 0 \end{aligned} \tag{36}$$

Let $S_A(t)$ denote the size of A, we have

$$S_A(t) = 2t \times 3^{n-t} - 1 \tag{37}$$

Let $Mean$ denote the average number of lightpaths passing through each unidirectional link in A, thus we have

$$Mean = C_A/S_A \tag{38}$$

Let $M(t)$ denote the lower bound of the backup number of wavelengths to maintain the all-to-all routing if the failed link is in the t^{th} dimension and $M = \max(M(t))$, thus we have

$$M(t) = \lceil Mean \rceil = \lceil \frac{3^{t-1}}{2t - 1/3^{n-t}} \rceil \quad (39)$$

where t is in $[1, n]$ thus $M(t)$ achieves its maximum when $t = n$, i.e.,

$$M = M(n) = \lceil \frac{3^{n-1}}{2n-1} \rceil \quad (40)$$

Let BW be the number of backup wavelengths for one link failure. Then

$$BW \geq \lceil \frac{3^{n-1}}{2n-1} \rceil \quad (41)$$

As shown in (40), the maximum backup number of wavelengths is needed when l_{ij} is in n^{th} dimension and node j is the destination of the 3^{n-1} affected lightpaths.

Since 3^{n-1} lightpaths will pass along l_{ij} if it is not failed, these lightpaths have to pass along the rest of $(2n-1)$ ending links to reach their destination node j. We evenly distribute the 3^{n-1} lightpaths among these adjacent unidirectional links, thus the number of backup wavelengths is $\lceil 3^{n-1}/(2n-1) \rceil$, which is consistent with (40).

The same results can be obtained for the case that the failed link is from the starting links of the source node p, which the maximum number of backup wavelengths is needed when l_{ij} is in the first dimension and node i is the source of the 3^{n-1} affected lightpaths.

4.2 Multiple Links Failures

As shown in Section 4.1, backup wavelengths can guarantee all-to-all routing in G_3^n if they could satisfy the worst case, i.e., the failed link is in the 1^{st} or the n^{th}dimension. Similarly, for the case of h links fails simultaneously, where $h > 1$, the worst case is that all the failed links are from the same set of starting or ending links of a node.

We assume that the h failed links are from the set of starting links of node i. If the failed link denoted by l_{ij} is in the t^{th} dimension, thus 3^{n-t} lightpaths, denoted by $T(t)$, starting from i are affected hence need to reassigned wavelengths. In the n-dimensional torus, a node has two starting links in one dimension, thus these $3^n - 1$ lightpaths from node i traverse these $2n$ starting unidirectional links in all dimensions, thus we have

$$3^n - 1 = 2(\sum_{t=1}^{n} T(t)) \quad (42)$$

Let the dimensions of the h failed links are $d_1, d_2, \cdots, d_h$, respectively; $f(h)$ denote the number of the affected lightpaths, we have

$$f(h) = \sum_{j=1}^{h} T(d_j) \quad (43)$$

As $T(t)$ is a monotonically decreasing function, $f(h)$ achieves its maximum when all the d_is achieve their minimum, thus we have

$$f(h) = \begin{cases} 3^n - 3^{n-h/2} & h \text{ is even} \\ 3^n + 2 \times 3^{n-(h+1)/2} & h \text{ is odd} \end{cases} \quad (44)$$

The backup wavelengths should support the $max(f(h))$ lightpaths to travel on the rest of unaffected $2n - h$ starting links of node i, thus we derive the minimum number of backup wavelengths for h links failures for the following two cases: let

Case 1: if h is even and $h \leq 2n$:

$$BW \geq \frac{3^n - 3^{n-h/2}}{2n-h} \quad (45)$$

Case 2: if h is odd and $\leq 2n$:

$$BW \geq \frac{3^n + 2 \times 3^{n-(h+1)/2}}{2n-h} \quad (46)$$

For $h \geq 2n$, one node is isolated from the network as all its $2n$ starting or ending links are unavailable, thus the all-to-all routing is impossible to realize, which is not considered in this work.

In practical, network operator can take advantage of a centralized controller to detect the failure, compute the routing and backup wavelength assignment algorithm as well as running our GAR algorithm to realize all-to-all routing when building the all-optical network.

5. RELATED WORK

Many previous works investigated the routing problem in all-optical bidirectional ring and torus networks [3, 4, 5, 17]. Ellinas et al. [18] showed that the number of wavelengths needed to support all-to-all routing over an unidirectional ring network is $N(N-1)/2$. Additionally, many previous works studied some specific graphs such as hypercubes [5] and trees of rings [19]. Pascu et al. [20] leveraged the tap-and-continue capability of the nodes in the optical network to address the all-to-all routing problem in arbitrary topologies. Narayanan et al. [21] developed an approximation algorithm to solve the all-to-all routing problem in chordal rings of degree 4. The number of wavelength needed is at most 1.006 times the theoretical minimal number of wavelengths. Beauquier al et. [19] discussed the all-to-all problem in the symmetric directed trees of rings. Gargano et al. [4] developed another method to prove that the minimum number of wavelengths is achievable in ring network. Multi-hop model [2] in lines, rings, 2D square tori, 3D square tori and complete binary trees has been well studied in [22, 23, 24, 25], but we only consider single-hop model in this paper, thus multi-hop model can be a research direction in our future works.

6. CONCLUSIONS

In this paper, we revisit the all-to-all routing problem in all-optical networks, which is well-studied by previous works, but they failed to propose a concrete routing and wavelength assignment algorithm to achieve the theoretical lower bound of the number of wavelengths. Therefore, we first develop a novel RWA algorithm, i.e., GAR, to solve the all-to-all routing in bidirectional ring network with odd nodes or even nodes, which achieves the lower bound of number of wavelengths needed to support the all-to-all routing. Second, we showed that the number of wavelengths to realize all-to-all routing in any torus topology is not less than $N/3$. Among those topologies, an n-dimensional hypersquare torus with side 3 on each dimension is shown to achieve the value, $N/3$, and a GAR-like routing and wavelength assignment algorithm is proposed to solve all-to-all routing over that topology. Finally, we address one/multiple links failure problem over that topology, i.e., we derive the lower bound of the number of backup wavelengths to maintain the all-to-all communication if one/multiple links fail over an n-dimensional hypersquare torus with side 3.

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